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SYNCHRONIZATION AND CONTROL IN A CELLULAR NEURAL NETWORK OF CHAOTIC UNITS BY LOCAL PINNINGS†

S. JANKOWSKI

Institute of Electronic Fundamentals, Warsaw University of Technology, Nowowiejska 15/19, 00–665 Warsaw, Poland

A. LONDEI

Department of Electronic Engineering, Università di Roma 'La Sapienza', Via Eudossiana 18, I-00184 Rome, Italy

A. LOZOWSKI

Department of Electronic Engineering, University of Louisville, Louisville, KY, U.S.A.

AND

C. MAZUR

Institute of Electronic Fundamentals, Warsaw University of Technology, Nowowiejska 15/19, 00–665 Warsaw, Poland

SUMMARY

We present a new technique for controlling the behaviour of a large system composed of chaotic units by using only a few control units referred to as pinnings. Our model can be regarded as an extension of cellular neural networks to chaotic cells, in this paper described by Lorenz equations, locally coupled by identical connections. The network is of moderate size, 27×27. By tuning the connection strength D, a large variety of global behaviours can be obtained: from fully turbulent to fully coherent spatiotemporal states. In between the system exhibits unstable partial synchronization. We show that by using one (or only a few) unit(s) controlled on a chosen unstable periodic orbit by the standard method of Ott, Grebogi and Yorke (OGY), the global dynamics can be substantially changed: all units tend to obey periodic dynamics. By appropriate placement of pinnings the spatiotemporal state of the network can be ordered and shaped.

1. INTRODUCTION

The spatiotemporal behaviour of complex systems has recently become an area of interest in the field of non-linear dynamics. The cellular neural network (CNN),[1] utilized as an analogue supercomputer performing image-processing tasks, is a simple example of such a system. There are several extensions to the basic CNN model, including a locally coupled lattice of chaotic oscillators which has been widely investigated.[2–4] A CNN composed of Chua circuits and employed to model coherent wavy behaviour in a large chaotic system has been introduced in Reference 2. In this model the desired spatiotemporal structures can be obtained as the network response to appropriately chosen initial conditions. Other authors have usually concentrated on coupled map lattices in order to work out various examples of chaotic neural networks that are able to perform chaotic dynamics without direct relevance to neurobiology.[5] Chaos allows the network to shift robustly from one complex activity pattern to another in response to small changes in input. Controlling chaos in large systems becomes an important engineering problem; potential applications include information processing by using spatiotemporal periodic orbits.[6]

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The problem of controlling the total system by modulating very few degrees of freedom has been addressed in Reference 7. Control which allows extraction of a given unstable reference orbit embedded in the unperturbed system dynamics can be achieved by placing local feedback pinnings in space. Although the question as to how dense the pinning locations in the lattice grid should be for effective control is a matter of study itself, any periodic pinnings are able to affect the system dynamics and eventually suppress chaos.[8] However, the pinning dynamic performance is of crucial importance in the control process.

The objective of the present paper is to introduce a new concept of controlling the spatiotemporal behaviour in large two-dimensional chaotic flow systems. We present a CNN composed of units described by the Lorenz equations, interacting locally by identical connections. We have chosen Lorenz-type units owing to their well-known dynamics which belong to the generalized Chua family of chaotic oscillators.[9] We attempt to control the spatiotemporal behaviour of the network by using a few unilateral (non-feedback) local pinnings embedded in the CNN grid. The pins are stabilized by the standard OGY method on a chosen unstable periodic orbit to match dynamically the remaining network units. The aim of this control is to suppress chaos and obtain partial temporal synchronization in the CNN, considered as activity pattern formation.

2. HOMOGENEOUS 2D CHAOTIC FLOW

We consider first a chaotic flow lattice composed of locally coupled non-linear oscillators described by the state vector $\vec{v}$. Here $\vec{v}$ is a continuous function of time described by the differential equation

$$\frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = R(\vec{v}) + D\nabla^2\vec{v} \tag{1}$$

where $R(\vec{v})$ denotes the internal oscillator equations, ∇^2 represents the local interaction pattern and D is a coupling constant related to the connection strengths. The parameter D is a real number that can be varied from zero to an arbitrary maximum value. Equation (1) can be approximated by the simple structure of a CNN.[10] We have investigated the Lorenz-type[11] chaotic CNN model. The terms on the right side of equation (1) are described by

$$R(\vec{v}) = R\left(\begin{bmatrix} \vdots \\ x_{ij} \\ y_{ij} \\ z_{ij} \\ \vdots \end{bmatrix}\right) = \begin{bmatrix} \vdots \\ \sigma(y_{ij} - x_{ij}) \\ rx_{ij} - x_{ij}z_{ij} - y_{ij} \\ -bx_{ij} + x_{ij}y_{ij} \\ \vdots \end{bmatrix}, \quad i, j = 1, \ldots, n \tag{2}$$

$$\nabla^2\vec{v} = \sum_{k=i-1}^{i+1} \sum_{l=j-1}^{j+1} A_{ij,kl}\, y_{kl} \tag{3}$$

Here σ, r and b are the Lorenz system parameters such that the network units perform chaotic oscillations when not coupled. We have chosen $\sigma = 16$, $r = 45{\cdot}92$ and $b = 4$. The dimension of $\vec{v}$, $R(\vec{v})$ and $\nabla^2\vec{v}$ is $3 \times n \times n = 3 \times N$, where N is the number of coupled oscillators. Using the notion of the CNN, the network templates A, B and the current I are expressed as

$$A_{ij} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \qquad B = I = 0 \tag{4}$$

Thus the network is a two-dimensional chaotic CNN. Toroidal boundary conditions have been used in this model.

The network described by (1) is fully homogeneous, so identical cells are coupled by identical bonds. It is significant that owing to interaction between units, the whole system can oscillate in an organized or coherent mode even though each oscillator embedded in the network behaves chaotically. The tendency to organization is related to the diffusion coefficient D. Hence D can act as a control parameter for chaos suppression. The spatiotemporal behaviour of the network can be described by the generalized variance V defined as

$$V = \lim_{t\to\infty} \sqrt{\left(\frac{\sum_{k=1}^{N} \sum_{j=k}^{N} [x_k(t) - x_j(t)]^2}{\binom{N}{2}} \right)} \tag{5}$$

Quantity (5) is related to the coupling strength D and the network size n. V is bounded between zero, denoting global synchronization, and some maximum value $V_{\max}$ indicating lack of correlation between units. Various dynamic modes achieved for these parameters can be recognized as phase states of the system corresponding to different values of V, as shown in the phase diagram of Figure 1. Generally, for a certain size n, three qualitatively different dynamic modes (phases) are available in the network by tuning the coupling strength D. These modes correspond to regions A, B and C found in the phase diagram. In the turbulent phase (region A), arising for weakly coupled cells, no synchronization between cells is observed. As shown in Figure 2(a), each oscillator performs Lorenz dynamics independently of the others, so there is no correlation between cells and no regularity may be observed in a 3D view of the network state.

An increase in D forces the network to organize (Figure 2(b)). In the organized phase, typical spontaneous behaviour is observed. For a specific moment of time, neighbouring cells have states close to each other, so the wave presented in a 3D view appears to be smooth. Although the cells tend to synchronize owing to the influence of couplings, there exist spontaneous bursts of the wave established. For large enough values of D, global synchronization of all cells emerges, as shown in Figure 2(c). Every oscillator performs the same Lorenz activity, so the 3D view of the network state presents a plane moving up and down with time. We wish to work out a method of controlling the pattern formation process in the network without making any assumptions on the initial conditions applied to the oscillators.

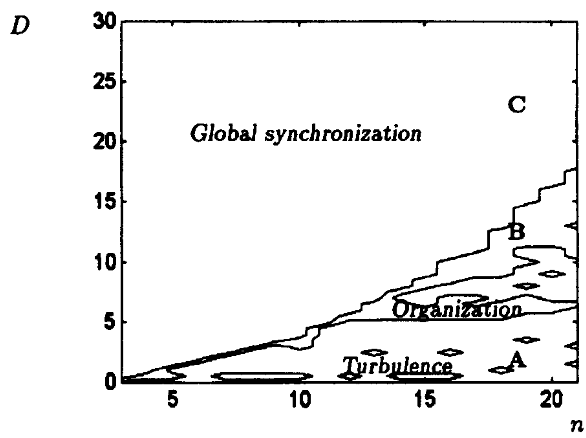


Figure 1. Phase diagram of chaotic CNN composed of $n \times n$ cells. D is the diffusion coefficient. The contours correspond to values of normalized variance $V = 3$ and 22

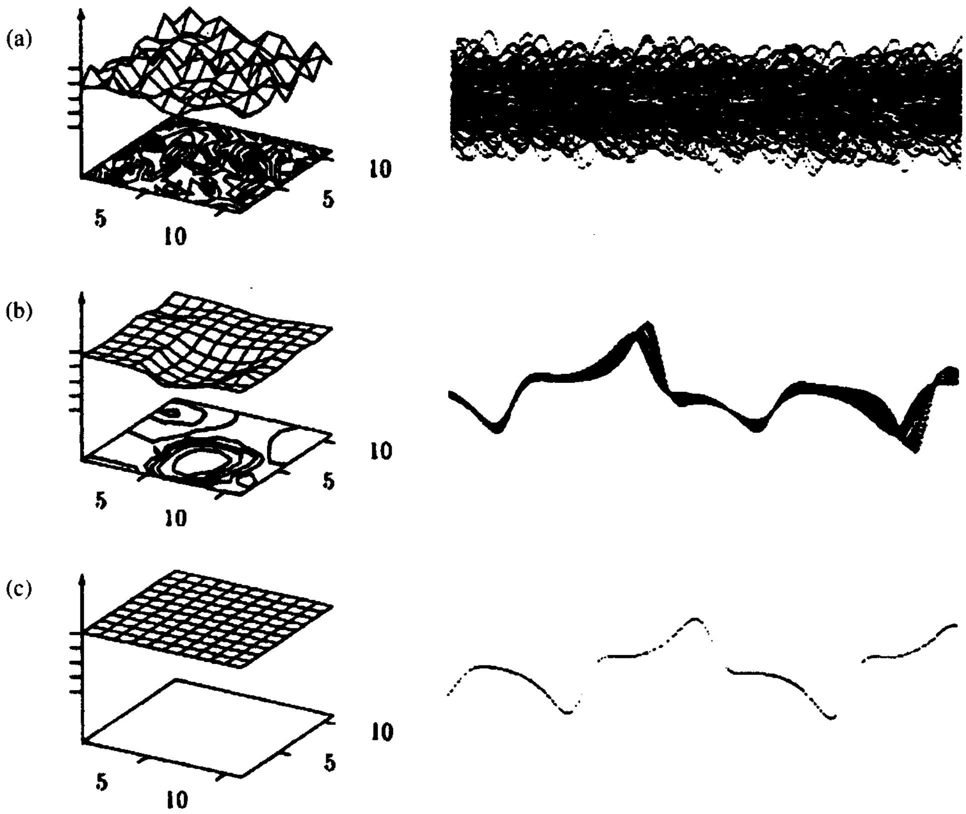


Figure 2. Global plots of CNN dynamics (number of cells, 11 × 11): (a) turbulent; (b) partially organized; (c) synchronized

3. NON-FEEDBACK PINNINGS

We introduce the idea of using isolated oscillators embedded in the network in order to organize the pattern formation process. They act as controlling pinnings in the net grid, affecting the dynamics in the ordered phase. The pinnings are numerous oscillators with unilateral coupling connections. The pinnings are not affected by the states of their neighbours, since the connection strengths leading from the neighbours to the specific pinning are of zero value independently of the diffusion coefficient D. Thus the pinnings are autonomous Lorenz oscillators. At the same time the connection strengths leading in the opposite direction are equal to D. The pinnings are located regularly in the net grid. An example of the location of autonomous pinnings placed in the middle of each quarter of an 11 × 11 (square) grid is shown in Figure 3. The neighbours tend to follow the pinning behaviour with respect to the diffusion coefficient D, which is chosen from the region corresponding to the ordered phase. The waves are formed in a spontaneous dynamical process, but defects formed in the network grid cause regularity in the created shapes. As shown in the contour plots of Figure 3, the pinnings seem to be the centres of local waves, while the entire wave shape remains smooth.

We will now study the control technique using synchronized and periodic pinnings. In the present approach we use a few periodic pinnings in order to control the pattern formation process. The pinnings are stabilized, using the OGY method, on a certain unstable periodic orbit that is embedded in the Lorenz dynamics performed by each unit when not coupled. This fact ensures the similarity between dynamics of chaotic and periodic units. Moreover, the OGY method enables us to choose between various types of orbits embedded in the Lorenz attractor. We have chosen an orbit of order one, which is the most frequently visited. The idea of the OGY method[12] is to change one of the system parameters in such a way as to obtain periodic oscillations. As a control parameter we chose the variable r in (2). The application of

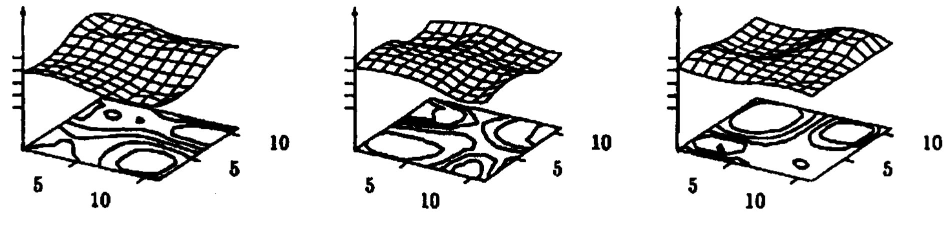


Figure 3. Dynamics of network with four autonomous seeds for subsequent moments of time (a)–(c)

the OGY method is as follows. Denoting X_n as a co-ordinate vector on the Poincaré plane of the nth-section upward trajectory, we obtain the discrete map

$$X_{n+1} = M(X_n) \tag{6}$$

The unstable equilibrium point, corresponding to the periodic orbit which should be stabilized, is described by the equation

$$X_F = M(X_F) \tag{7}$$

We do not know the exact position of the point X_F, but only the region to which it belongs. In a small neighbourhood of this point we can use a linear approximation of the map M by the Jacobian matrix A:

$$X_{n+1} = AX_n \tag{8}$$

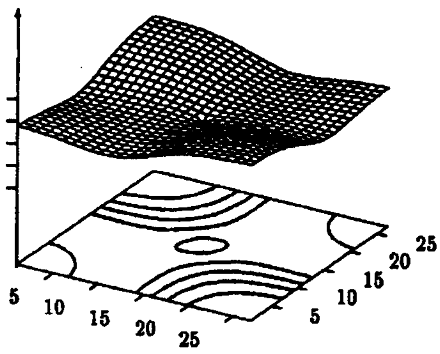
Having observed the behaviour of trajectories crossing the Poincaré plane near the equilibrium point, we can calculate all required parameters of the system. Using a least squares algorithm, we approximate components of the matrix A, eigenvectors, corresponding eigenvalues and the exact position of the equilibrium point. Using small changes in the of control parameter r, we can also find their influence on the system. Applying the control signal (Δr), we perturb the system in such a way that each successive point X_n is on the attracting direction, causing each successive X_n to be closer to the equilibrium point, while the system trajectory becomes closer to a periodic orbit.

4. CONTROL TECHNIQUE

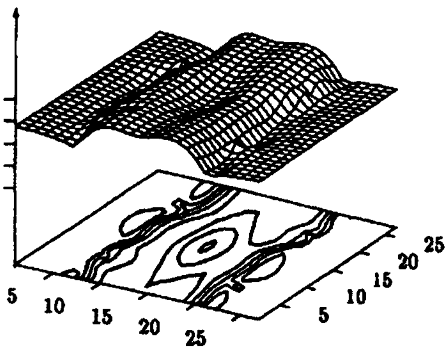
Using periodic pinnings located regularly in the CNN grid, we are able to control the network spatiotemporal states. The pinnings are made to oscillate either coherently or with shifted phases with arbitrarily chosen phase shift distributions. The distribution is obtained by copying state values from one OGY controlled unit to the other pinnings with positive or negative signs. Various types of patterns that are to appear during the control can be determined in this manner.

Since pattern formation is a dynamical process, the network state starting from some initial conditions tends to a certain type of behaviour that is relevant to the phase state dominant for the chosen coupling strength D. If D is assigned a value which belongs to the ordered phase on the phase diagram (Figure 1), the dynamical process converges to wavy oscillations which are recognized as a pattern. In order to improve the pattern formation process, phase transitions may be applied to the network. The procedure of such control is the following

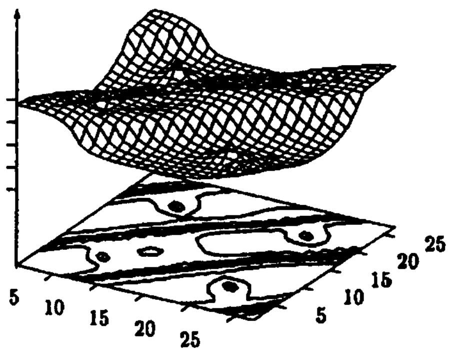
1. First the turbulent phase state of the network should be established by reducing the value of D. After a sufficiently long period of time to 'chaoticize' the network state, the unit oscillations become uncorrelated. In this mode it is possible to go through the basins of attraction of various final patterns that are available after transition into the ordered phase. This initial step is located in Figure 1 as point A.



(a)



(b)



(c)

Figure 4. Patterns formed in chaotic CNN using (a) one pinning placed in middle and (b), (c) four pinnings placed in middle of each quarter of CNN grid

2. Successively we increase the parameter D in order to move from point A into the globally synchronized region (point B). A rapid organization of the units into some well-defined patterns ruled by the pinnings is observed. This process tends to achieve the behaviour specific to that region and thus should be interrupted when a pattern is formed.
3. Finally we move the phase point into C (ordered) by decreasing the value of D. The pattern previously formed may remain stable in time; hence, by controlling a few elements in the network, we can achieve a global organization and the final pattern is frozen as a natural mode of the network.

In Figure 4, three different patterns are shown by plotting the value of every unit at a certain moment; they are not considered to be constant in time, but instead as moving in an oscillatory manner. Figure 4(a) is built by putting a single pinning in the centre of a 27×27 network and freezing the emergent pattern. Figures 4(b) and 4(c) are obtained by using four pinnings alternately in counterphase; these patterns are found for various moments of time in the turbulent region.

5. CONCLUSIONS

A non-feedback control method for large chaotic CNNs by using numerous pinnings placed at certain locations of the network grid has been presented. To our knowledge the present approach is a novel concept for controlling the spatiotemporal behaviour of Lorenz units with coupled lattice flow. By employing the standard OGY method, the controlling pinnings are stabilized on unstable periodic orbits extracted from the Lorenz attractor. Thus the controlling pinnings match dynamically the nature of the network units. Tuning the coupling parameter D is a suitable way of improving the control efficiency. By means of the introduced control method a chaos suppression and pattern formation process has been achieved. However, there is a need for an estimate of the spatiotemporal chaos in the area of interest. Also of importance is the investigation of transient chaos time in an on/off control regime. Considerations on this matter will be presented elsewhere.

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