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Synchronization and association in a large network of coupled Chua's circuits†

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This work presents the results of simulation of the fully connected networks of Chua's circuits mutually coupled by nonlinear conductances derived using the Hebbian learning rule. The network can be regarded as a generalization of the Hopfield neural network built up of chaotic units. Due to the space–time synchronization of units, the studied network exhibits the ability of pattern retrieval and decorrelation of complex input patterns.

1. Introduction

There is experimental evidence of intrinsic chaotic behaviour of neural systems (Skarda 1987). The role of coherent oscillations in the feature linking mechanism of a cat's visual cortex was explained by Eckhorn (1988). Many attempts have been made in order to use chaotic dynamics for storing information in artificial neural networks (e.g. Tsuda 1992).

As Chua's circuit (Chua 1990) is the best known model of an oscillator that exhibits very rich chaotic dynamics and very simple circuitry, we have chosen it as a basic component of our model network. We explore Chua's circuits with the double-scroll attractor. The reasons for using the double-scroll are twofold:

(1) the existence of two different states: the 'upper' and 'lower' scroll;
(2) the relatively long time of setting in one scroll.

The objective of this paper is to study the mechanism of global spatiotemporal coherence in the large, fully connected, network of Chua's circuits obtained by assigning the given pattern vectors to synchronization modes in a method analogous to the Hebbian learning rule (Hertz *et al.* 1991). This chaotic model also exhibits some other interesting properties, such as: pattern association, decorrelation of complex patterns and fluctuating attention.

2. Fully connected network of Chua's circuits

2.1. *Network structure*

The considered network, as presented in Fig. 1, is composed of N fully connected identical Chua's circuits described by a set of differential equations

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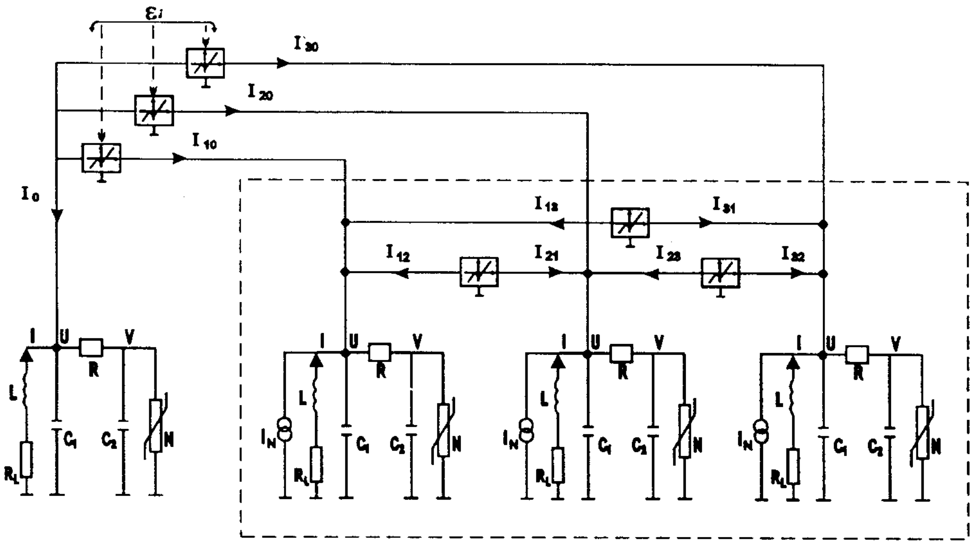

Figure 1. Fully connected network of coupled Chua's circuits ($N=3$).

$$C_1 \frac{dU_i}{dt} = \frac{1}{R}(V_i - U_i) + I_i \sum_{j=1, j\neq i}^{N} I_{ij} + I_{io} + I_{Ni} \tag{1}$$

$$C_2 \frac{dV_i}{dt} = \frac{1}{R}(U_i - V_i) - f(V_i) \tag{2}$$

$$L \frac{di_i}{dt} = -U_i - I_i R_L \tag{3}$$

The values of circuit parameters are chosen in order to ensure the double-scroll attractor (Ogorzałek 1993).

The whole network is controlled by the outer Chua's circuit by unilateral coupling elements. The control circuit operates in the autonomous mode because the input current $I_o = 0$. The controlling coupling elements define the binary input pattern vector by mapping the vector components onto the signs of the input elements. In order to approach the physical reality we introduce a current noise source I_N of small RMS value. The noise is supplied in parallel to each Chua's circuit.

2.2. *Couplings*

The application of a double-scroll attractor for storing binary patterns requires the possibility of synchronization states as well as anti-synchronization ones. The synchronization states can be reached if the current I_{ij} flowing through the coupling elements tends to zero under the condition that the potentials V_i and V_j are equal to each other and, consequently, both circuits approach the same dynamical behaviour. The anti-synchronization state may occur if the potentials V_i and V_j have exactly opposite values in the function of time in the absence of coupling current I_{ij} flow.

The conditions for both kinds of synchronization cannot be satisfied using simple linear conductances as coupling elements; therefore, we propose the following model of a four- terminal coupling element

$$I_{ij} = g_{ij}\left[U_j + \operatorname{sign}(g_{ij}) \cdot U_i \left(\left| \frac{U_j}{U_i} \right| - \left| \frac{U_j}{U_i} + \operatorname{sign}(g_{ij}) \right| \right) \right] \quad (4)$$

$$I_{ji} = g_{ji}\left[U_i + \operatorname{sign}(g_{ji}) \cdot U_j \left(\left| \frac{U_i}{U_j} \right| - \left| \frac{U_i}{U_j} + \operatorname{sign}(g_{ji}) \right| \right) \right] \quad (5)$$

As shown in fig. 2, the coupling element has piecewise-linear characteristics, which enables coupling current I_{ij} to be zero if $U_i = U_j$ (synchronization) and if $U_i = -U_j$ (anti-synchronization).

2.3. *Hebbian learning*

The fully connected network of N chaotic oscillators has 2^N either stable or unstable equilibrium points. If we assume that stable equilibrium points are assigned to the desired binary pattern vectors $\{P_i\}^N$, $P_i = \pm 1$ then the considered network can play the role of an associative memory. The pattern vectors can be mapped onto the equilibrium points if the signs of vector components are related to the values of nonlinear coupling conductances g_{ij} following the Hebb learning rule (Tsuda 1992)

$$g_{ij} = g_o \cdot \left(\sum_{\mu=1}^{q} P_i^\mu P_j^\mu \right) \quad (6)$$

Thus, as in the Hopfield model, q uncorrelated and unbiased patterns can be stored. Therefore, the network dynamics are determined by the conductance matrix:

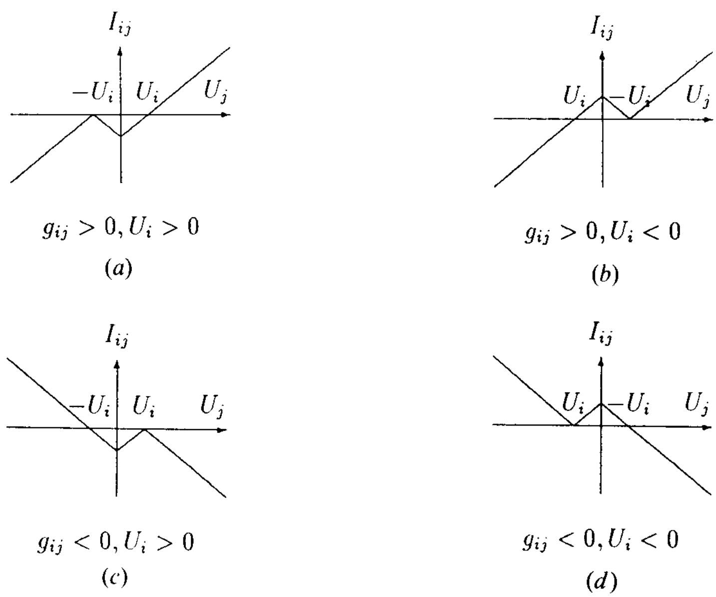

Figure 2. Nonlinear coupling conductances. Coupling current I_{ij} versus node potential U_j (the case $|g_{ij}| = 1$).

$G=[g_{ij}]^{N\times N}$, $g_{ij}=g_{ji}$. The orthogonal pattern vectors are identical to eigenvectors of the matrix G.

3. Decorrelations and association in the network

3.1. *Synchronization phenomenon*

The synchronization of two Chua's circuits coupled between the adequate nodes by conductances can be explained as approaching identical oscillations by the two circuits under consideration (Ogorzałek 1993) and is also observed in a large network of coupled oscillators. However, the synchronization state is interrupted by short desynchronization intervals caused by the positive Lyapunov exponent in those regions of the chaotic trajectory where the infinitesimal differences between states lead to fast desynchronization of circuits. This effect can be considered as a model of fluctuating attention, which is experimentally observed in biological nervous systems (Skarda and Freeman 1987, Wang *et al.* 1990).

The increase of the desynchronization probability can be achieved by the noise source, which is equivalent to the natural inhomogeneity of the network structure.

3.2. *Association and decorrelation*

The aim of Hebb's connectivity matrix was to create an associative memory capable of storing and retrieving binary patterns analogously to the Hopfield neural network. However, our network acts dynamically and there are not stationary states. Nevertheless, we found that the considered network is able to associate patterns presented in the input and reconstruct pattern distortion due to the information placed in Hebb's connections.

In the numerical experiments, we used the network of $N=16$ Chua's circuits. The learning set consisted of three uncorrelated and unbiased patterns $\{P_1, P_2, P_3\}$ (see Fig. 3(a)). The network was then presented with a new pattern P_0, correlated with

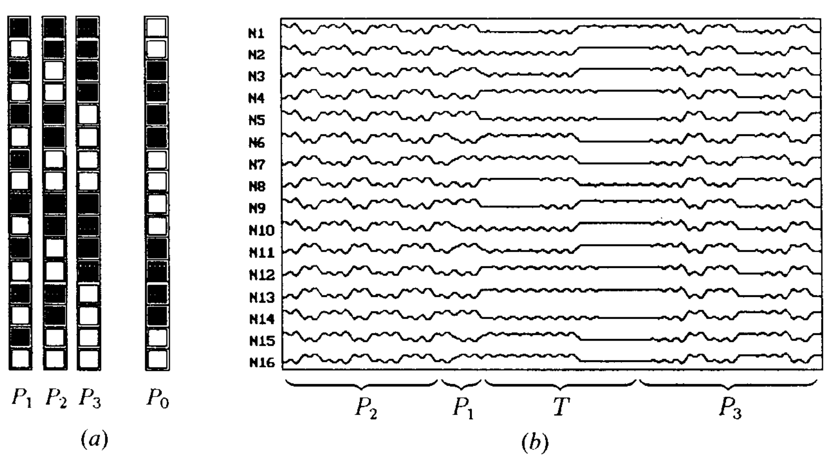

Figure 3. Association and decorrelation effect: (a) patterns; (b) signal time plot of 16 neurons of the network ($N1-N16$).

P_3 only. The network responded by generating a sequence of mutual synchronization states. All of the synchronization states correspond to the stored pattern which was correlated with the input.

Retrieving ability depends on the input signal level and the correlation between the stored and input pattern. As shown in Fig. 4, for arbitrary chosen, small input couplings of strength $k = g_{io}/g_o$, there exists a threshold value of correlation c_{03} which enables the network to distinguish which one of the patterns is currently being retrieved.

Suppose the input pattern P_0 is correlated with more than one pattern in the learning set. (In the simulations there was $c_{01} = c_{02} = c_{03}$, see Fig. 3.) Compared with neural associative memories, the difference is that this network produces a sequence of synchronization states that correspond to all of the stored patterns resembling the input. Thus, we achieve a decorrelation effect of the input pattern into components defined using the Hebbian learning rule.

The transition to another retrieved pattern occurs due to the relatively large value of the local Lyapunov exponents, as can be observed in the decorrelation interval T in Fig. 3.

Furthermore, there is a possibility of distinguishing similarities between the components. As shown in Fig. 5, the more frequently each stored pattern appears, the greater is its mutual correlation with the input pattern. This effect depends on input connectivity strength k. It disappears for k too small (all patterns appear with the same frequency) as well as for k too large.

4. Conclusions

The network presented in this paper is characterized by individual oscillatory units, with a global synchronization mode, for storing information. The main observation can be summarized as follows.

(1) The dynamics of the network of coupled Chua's circuits contain temporary states of synchronization between circuits, which depend on the coupling connectivity matrix.

(2) The network can perform the task of an associative memory, which recalls binary patterns. They may be stored in the connectivity derived by Hebbian

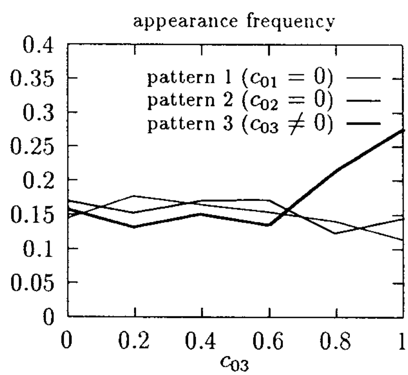

Figure 4. Retrieving ability (association of one pattern). Three stored patterns are correlated with the input as 0, 0 and c_{03} respectively. Input strength k is 0·1. The plots present the appearance frequency of each pattern versus mutual correlation c_{03}.

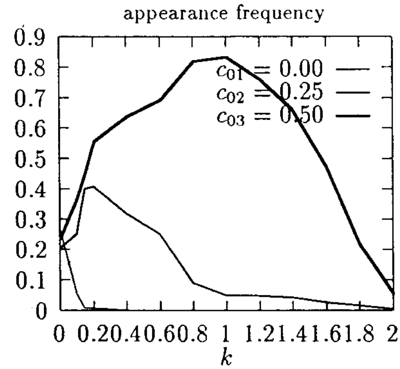

Figure 5. Decorrelation effect. Three patterns appear at the net outputs due to Hebbian learning. Mutual correlations between them and an input pattern are c_{01}, c_{02} and c_{03}. The three plots present the appearance frequency of stored patterns versus input coupling strength $k = g_{io}/g_o$.

learning. Hebb's coupling connections make synchronization states corresponding to the stored patterns.

(3) Since there are no satisfactory states in the dynamics of the network it is possible to achieve an effect of pattern decorrelation. When presented with a pattern similar to some of the stored patterns, the network produces a sequence pointing to those patterns that resemble the input. Mutual correlations between the input and the learning set may be distinguished by measuring the appearance frequency of each pattern in the sequence.

Acknowledgments

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