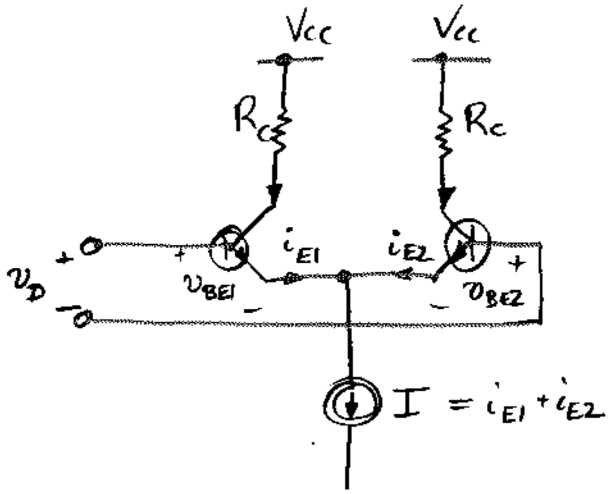
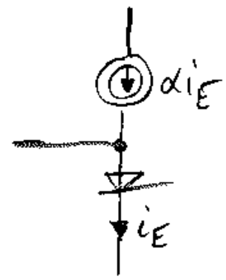
LARGE-SIGNAL OPERATION OF THE BJT DIFF. PAIR



using the T model.



$$i_{E1} = I_{SE}\, e^{\frac{v_{BE1}}{v_T}}$$

$$i_{E2} = I_{SE}\, e^{\frac{v_{BE2}}{v_T}}$$

"−" $$i_{E1} - i_{E2} = I_{SE}\left(e^{\frac{v_{BE1}}{V_T}} - e^{\frac{v_{BE2}}{V_T}}\right)$$

"+" $$i_{E1} + i_{E2} = I_{SE}\left(e^{\frac{v_{BE1}}{V_T}} + e^{\frac{v_{BE2}}{V_T}}\right)$$

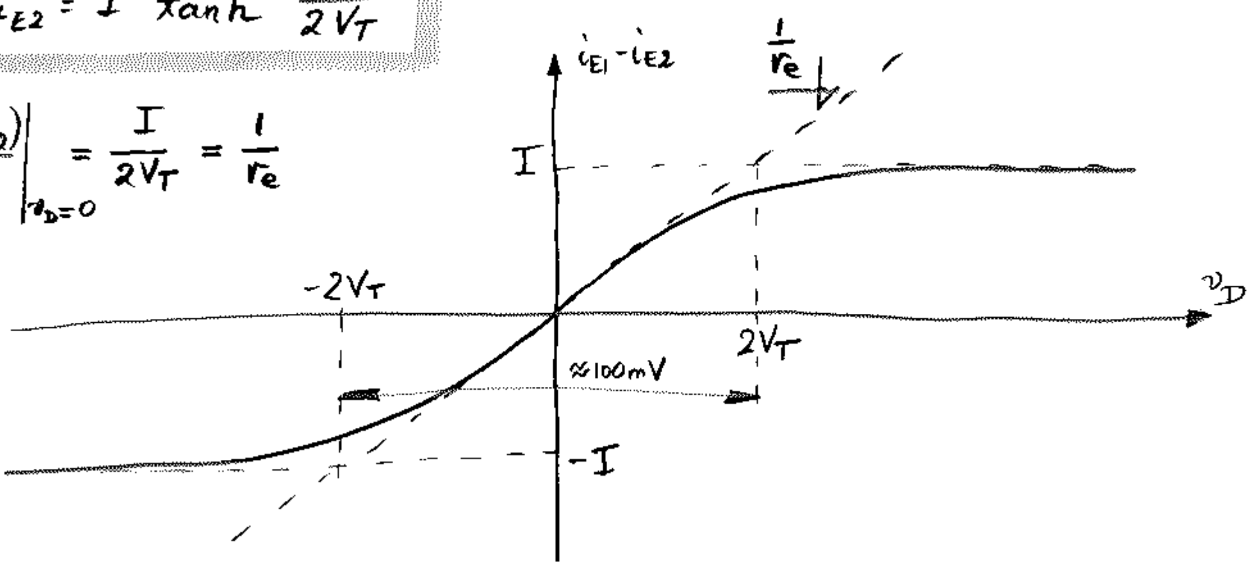
Note:

$$\frac{e^x - e^y}{e^x + e^y} = \frac{e^{\frac{-x-y}{2}}}{e^{\frac{-x-y}{2}}}\,\frac{e^x - e^y}{e^x + e^y} = \frac{e^{\frac{x-y}{2}} - e^{-\frac{x-y}{2}}}{e^{\frac{x-y}{2}} + e^{-\frac{x-y}{2}}} = \tanh\frac{x-y}{2}$$

$$\frac{i_{E1} - i_{E2}}{i_{E1} + i_{E2}} = \tanh\frac{v_{BE1} - v_{BE2}}{2V_T}$$

$$i_{E1} - i_{E2} = I \tanh\frac{v_D}{2V_T}$$

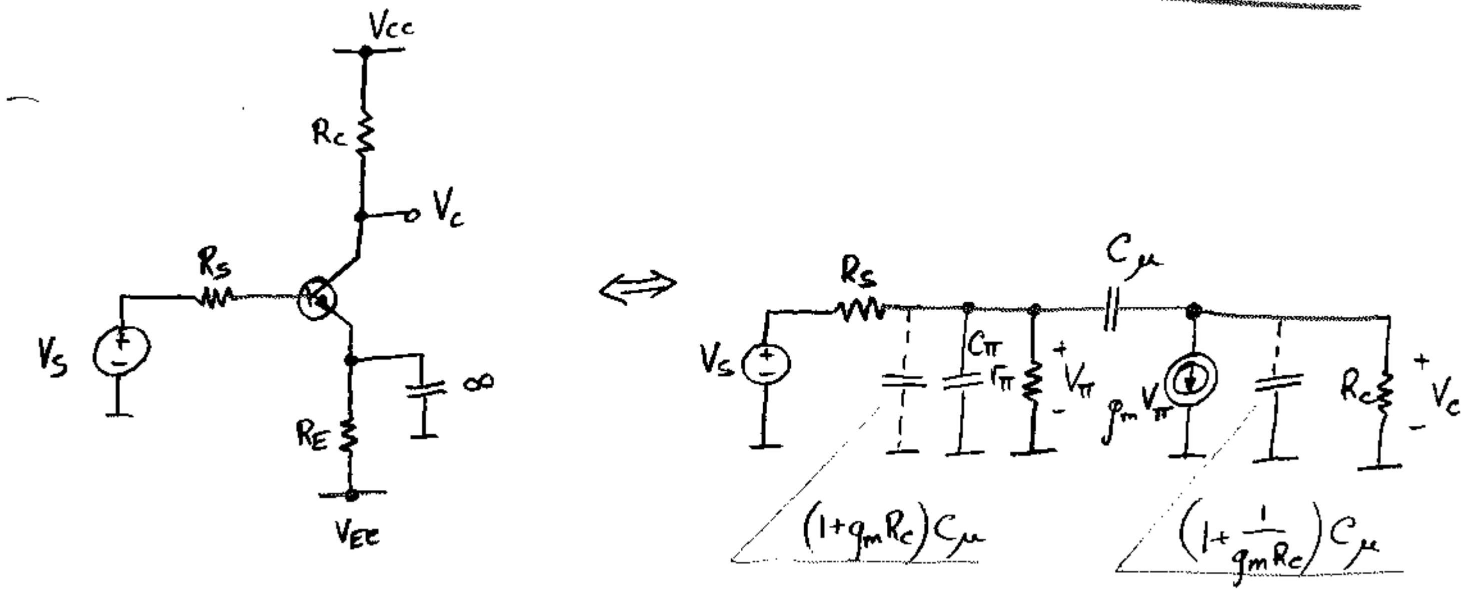
$$\left.\frac{d(i_{E1} - i_{E2})}{dv_D}\right|_{v_D=0} = \frac{I}{2V_T} = \frac{1}{r_e}$$



or

$$\left.\frac{d(i_{C1} - i_{C2})}{dv_D}\right|_{v_D=0} = \frac{\alpha}{r_e} = g_m$$

HIGH FREQUENCY RESPONSE OF A CE AMPLIFIER (BJT)



Mid-band voltage gain:

$$V_\pi = V_c \frac{r_\pi}{R_s + r_\pi} \quad , \quad V_c = -g_m V_\pi \cdot R_c = \underbrace{-\frac{r_\pi}{R_s + r_\pi} g_m R_c}_{A_{vo}} V_s = -\underbrace{\frac{R_s \| r_\pi}{R_s}}_{\frac{1}{a}} \underbrace{g_m R_c}_{A} V_s$$

High freq. response:

$$f_{H1} = \frac{1}{2\pi (R_s \| r_\pi)\left[(1+g_m R_c) C_\mu + C_\pi\right]}$$

$$f_{H2} = \frac{1}{2\pi R_c \left(1 + \frac{1}{g_m R_c}\right) C_\mu}$$

$$f_H = \frac{f_{H1} f_{H2}}{f_{H1} + f_{H2}}$$

Gain-bandwidth product:

$$\frac{1}{A_{vo} \, 2\pi f_H} = \left[(R_s \| r_\pi)\left[(1+g_m R_c) C_\mu + C_\pi\right] + R_c \left(1 + \frac{1}{g_m R_c}\right) C_\mu\right] \frac{R_s}{R_s \| r_\pi} \frac{1}{g_m R_c} =$$

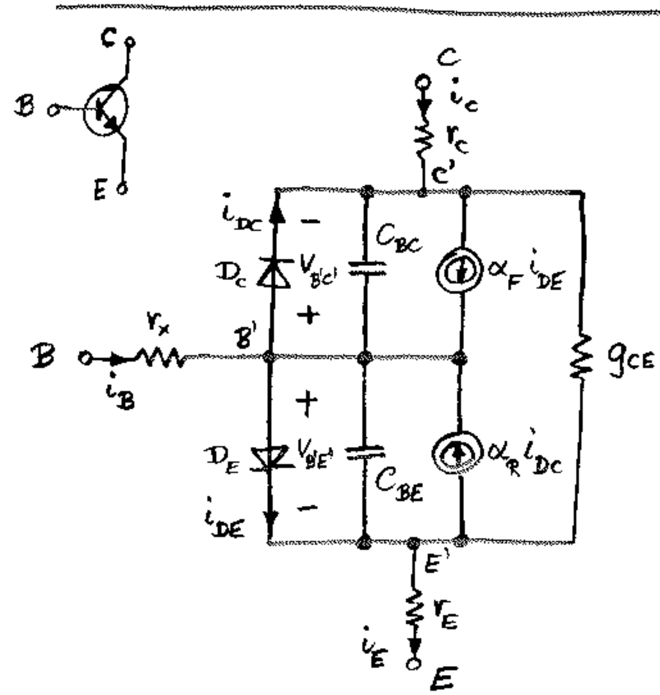
$$= \left(\frac{R_s}{a}\left[(1+A) C_\mu + C_\pi\right] + R_c\left(1 + \frac{1}{A}\right) C_\mu\right) \frac{a}{A} =$$

$$= R_s \left(1 + \frac{1}{A}\right) C_\mu + \frac{1}{A} R_s C_\pi + \frac{a}{A} R_c \left(1 + \frac{1}{A}\right) C_\mu =$$

$$= R_s C_\mu + \frac{1}{A} R_s (C_\mu + C_\pi) + R_c \frac{a}{A}\left(1 + \frac{1}{A}\right) C_\mu$$

$$(GB) = \frac{1}{2\pi} \frac{1}{R_s C_\mu + \frac{1}{A}\left[R_s (C_\mu + C_\pi) + a R_c \left(1 + \frac{1}{A}\right) C_\mu\right]}$$

THE COMPLETE EBERS-MOLL MODEL OF A BJT:



$$i_{DE} = I_{SE}\left(e^{\frac{V_{B'E'}}{n_F \varphi_T}} - 1\right)$$

$$i_{DC} = I_{SC}\left(e^{\frac{V_{B'C'}}{n_R \varphi_T}} - 1\right)$$

with $$g_{BE} = \frac{d\, i_{DE}}{d\, V_{B'E'}} = \frac{i_{DE} + I_{SE}}{n_E \varphi_T}$$

$$g_{BC} = \frac{d\, i_{DC}}{d\, V_{B'C'}} = \frac{i_{DC} + I_{SC}}{n_C \varphi_T}$$

$$g_{CE} = G_Z + N_G\left(g_{BE} + g_{BC}\right)$$

$$C_{BE} = \alpha_F \tau_F g_{BE} + \frac{C_{jeO}}{\left(1 - \frac{V_{B'E'}}{V_{oe}}\right)^{m_E}}$$

$$C_{BC} = \alpha_R \tau_R g_{BC} + \frac{C_{jcO}}{\left(1 - \frac{V_{B'C'}}{V_{oc}}\right)^{m_C}}$$

<u>Example</u> set of parameters of a BJT:

$I_{SE} = 0.02$ pA (emitter saturation current)

$I_{SC} = 0.0264$ pA (collector saturation current)

$\varphi_T = \frac{kT}{q} \approx 0.026$ V (thermal potential of electron)

$n_F = 1$ (emitter emission coefficient)

$n_R = 1$ (collector emission coefficient)

$\alpha_F = 0.99$ (forward scale factor)

$\alpha_R = 0.75$ (reverse scale factor)

$G_Z = 0$ (Early's parameter)

$N_G = 0.00025$ (Early's parameter)

$\tau_F = 0.2$ ns (forward transit time)

$\tau_R = 6.3$ ns (reverse transit time)

$C_{jeo} = 5$ pF (emitter junction unbiased cap)

$C_{jc0} = 4$ pF (collector junction unbiased cap)

$V_{oe} = 0.8$ V (emitter junction built-in volt.)

$V_{oc} = 0.8$ V (collector junction built-in volt.)

$m_E = 0.333$ (emitter Schottky's grading coeff.)

$m_C = 0.333$ (collector Schottky's grading coeff.)

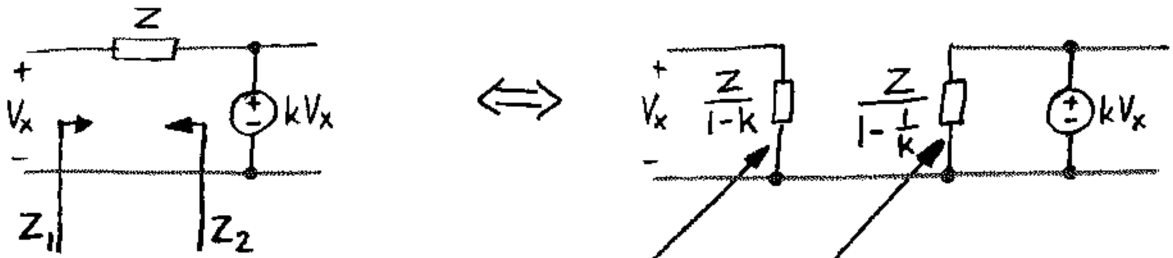
$r_x = 50\,\Omega$ (zero-bias base ohmic resistance)

$r_E = 0\,\Omega$ (zero-bias emitter ohmic resistance)

$r_C = 0\,\Omega$ (zero-bias collector ohmic resistance)

The Miller effect

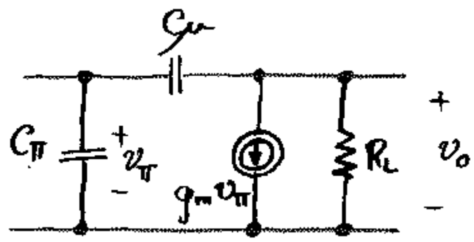
The following two circuits are equivalent:



$$Z_1 = \frac{V_x}{\frac{V_x - kV_x}{Z}} = \frac{Z}{1-k}$$

$$Z_2 = \frac{kV_x}{\frac{kV_x - V_x}{Z}} = \frac{Z}{1-\frac{1}{k}}$$

This fact simplifies the analysis of transistor circuits:



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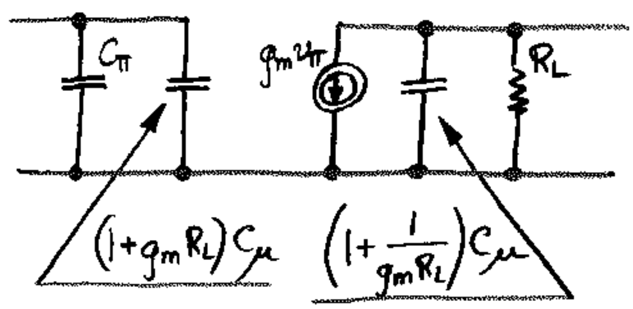

The mid-frequency gain is:

$$k = \frac{v_o}{v_\pi} = \frac{-g_m v_\pi \cdot R_L}{v_\pi} = -g_m v_\pi$$

$$Z_\mu = \frac{1}{sC_\mu} \quad \rightarrow \quad Z_1 = \frac{Z_\mu}{1+g_m R_L} = \frac{1}{s(1+g_m R_L)C_\mu}$$

$$Z_2 = \frac{Z_\mu}{1+\frac{1}{g_m R_L}} = \frac{1}{s\left(1+\frac{1}{g_m R_L}\right)C_\mu}$$